Atmospheric Turbulence Considerations for Future Aircraft Designed to Operate at Low Altitudes

J. D. GAULT* AND D. E. GUNTER JR.†
The Boeing Company, Wichita, Kansas

A research program (LO-LOCAT) is presently in progress to investigate the characteristics of atmospheric turbulence at low altitudes. A significant quantity of gust-velocity and meteorological data are being measured by instrumented aircraft. Resultant information will be incorporated in design criteria for those aircraft slated for operation in the next two decades. One of the items required for these criteria, and one objective of the program, is an analysis of atmospheric turbulence spectra. Results obtained thus far, regarding this aspect of the research, are discussed. Good correlation has been found between experimentally determined spectra and mathematical expressions as advanced by von Kàrmàn, and Lumley and Panofsky. Conversely, the experimental data have shown poor agreement with expressions developed by Dryden and by Lappe. This finding is pertinent, since the Dryden expressions have been used extensively in the past and are, in fact, recommended by some agencies for use in present aircraft design. An analysis of turbulence scale lengths associated with 1200 of the experimentally measured spectra is included. Trends in scale length variation with associated geophysical characteristics and gust-velocity statistics are shown. These data give an indication of the low-altitude atmospheric turbulence model which is evolving from the LO-LOCAT program.

Nomenclature

C = a factor appearing in the longitudinal gust-velocity spectra expression suggested by Lumley and Panofsky
cpf = cycles/ft
k = spatial frequency, cpf
L = longitudinal, integral scale length of turbulence, ft
PSD = power spectral density, power spectrum, spectrum
Φ = one-dimensional gust-velocity power spectral density, $(\text{fps})^2/\text{cpf}$
σ = standard deviation

Subscripts

D = Dryden equations
K = von Kàrmàn equations
L = Lappe equation
P = Panofsky and Lumley equation
T = truncated
t = time series
u = from longitudinal gust-velocity time series
v = from lateral or vertical gust-velocity time series
1 = lower frequency limit of truncated spectrum
2 = upper frequency limit of truncated spectrum

Introduction

OPERATIONAL requirements of the 1980's have indicated aircraft designs characterized by high speed and structural flexibility. Military aircraft missions will include, as in the past, low-altitude contour flight to avoid radar detection. As a result, atmospheric turbulence has become a major consideration in aircraft design. In the case of military aircraft, detailed knowledge of the environment at low altitudes has become particularly important.

Load factors arising as a consequence of atmospheric turbulence have been considered in the structural design of aircraft for many years. In the design of modern aircraft, the discrete approach is now being supplemented by power spectral density (PSD) analysis techniques. Increased usage of the PSD techniques has, in turn, established the requirement for a more detailed model of atmospheric turbulence for usage in this approach.

A model for low-level atmospheric turbulence is being developed by The Boeing Company in cooperation with the United States Air Force during a research program designated LO-LOCAT. The program features improvements over past low-altitude turbulence studies including broad coverage of geophysical phenomena. The probability of encountering a given level of turbulence under various geophysical conditions is being evaluated. An additional purpose of the program is to analyze turbulence power spectra associated with these geophysical conditions. Results pertaining to the latter endeavor are discussed in this paper.

Data Acquisition

The LO-LOCAT atmospheric turbulence data analyzed were obtained during a 15-month period by four instrumented C-131 aircraft. The aircraft were located at different bases within the United States and were flown over test routes near these bases. The routes, located in New York, Kansas, Colorado, and California, were established to give the widest range possible in topographical and climatological conditions and were selected such that populated areas would be avoided. A test route consisted of eight straight legs, each of which was approximately 20 naut miles in length. The legs were traversed in the same direction on each flight.

The aircraft were flown by Air Force personnel and the resultant data shipped to Boeing for processing and analysis. Normally, data flights were scheduled every other day. This was varied as necessary when weather conditions, aircraft maintenance problems, or other factors interfered. On each flying day, three missions were scheduled; one at dawn, one at mid-morning, and one in the afternoon. Two specific altitudes were used for the gust data gathering portion of test flights. These altitudes were 250 and 750 ft above the terrain. The pilot followed the terrain contour, as closely as safety allowed, using a radar altimeter to maintain a constant absolute altitude. Only one altitude was used for any

Presented as Paper 68-216 at the AIAA Aircraft Design for 1980 Operations Meeting, Washington, D. C., February 12–14, 1968; submitted March 11, 1968; revision received June 24, 1968. Research supported by U. S. Air Force/SEG Contract AF33(615)-3724.

* Research Specialist. Associate Fellow AIAA.
† Flight Test Analyst.

flying day, and the test altitude was alternated on successive flight dates.

The test aircraft were instrumented to measure meteorological and gust-velocity data. Radar altimeters, doppler systems, radiometers, and outside air temperature (OAT) probes were the basic instrumentation used to record meteorological data. A hot-wire anemometer was installed on each aircraft to obtain gust-velocity information in the 5–500 cps frequency range. The primary frequency range of interest, from 0.04 to 10 cps, was sensed by instrumentation contained in a pressure-type gust probe mounted on the end of a nose boom. Additional instrumentation was located at the boom base (attitude and roll rate gyros). In the probe design, motion sensors were placed in close proximity to gust sensors. Instrumentation contained inside the gust probe sensed the following: sideslip and attack angles, vertical, lateral, and longitudinal accelerations, static and impact pressures, pitch and yaw rates.

Fine and coarse measurements were provided to improve resolution. To insure accuracy, the temperature of each transducer was maintained at 135°F with thermostatically controlled heaters. A solid-state narrow band frequency modulated system and one track of wide band frequency modulation was used for data recording.

The boom, on which the gust probe was mounted, was designed for optimization between airflow and vibration characteristics. The probe was placed far enough in front of the airplane to minimize fuselage influences on airflow. The boom was designed for maximum stiffness-to-weight ratio. To achieve these goals, a tapered wall thickness was incorporated in the design; the ratio of boom length to maximum fuselage diameter was 1.3.

Five and one-half minutes of information, recorded while flying over each leg, constituted a gust-velocity condition. The variation of gust velocity with time was computed from these data. In the calculations, airplane motion effects were removed giving three orthogonal, space oriented gust-velocity components. Power spectral densities of these components were then computed to establish representative spectra for various geophysical situations. High-pass numerical filtering of the gust-velocity time function was used to insure valid spectrum calculations, especially at the lower frequencies. Dynamic calibrations of all measurements used in the gust-velocity computation were employed.

The instrumentation system and data processing methods used for this program were developed to cope with the many difficulties normally encountered in turbulence measurement. During development of the Boeing gust probe, for example, it was found that measured turbulence spectra were being adversely affected by probe aerodynamics. Frequency response of the gust probe was evaluated, therefore, during wind-tunnel testing. The transfer functions thus obtained were compensated during gust-velocity calculations yielding considerable improvement in resultant spectra. Procedures and analysis methods used during the LO-LOCAT Program are discussed in detail in Ref. 1.

Turbulence Spectra

In recent years, the usage of power spectra techniques in aircraft design has continued to increase. Indications are that future design work will rely even more on this approach, requiring a detailed knowledge of the power spectral density of atmospheric turbulence in the design frequency range. Once this is known, aircraft response spectra may be computed using applicable transfer functions.

To define atmospheric turbulence spectra, LO-LOCAT experimentally measured spectra are being compared to a number of mathematical expressions. The analysis is accomplished on a category basis; i.e., the spectra are grouped according to the geophysical conditions under which they were obtained. These groupings consider terrain roughness, the altitude at which data were recorded, and atmospheric stability. Terrain roughness was determined by computing the standard deviation of the terrain profile of each test leg. Atmospheric stability (lapse rate) is computed from air temperature data, measured at the beginning of each gust-velocity condition, at 100 and 1000 ft absolute altitudes.

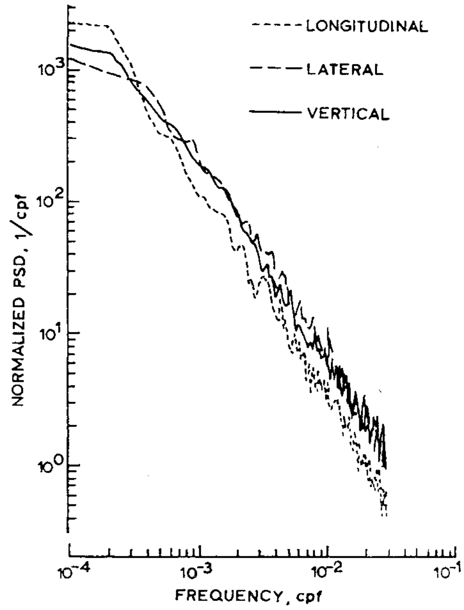

Fig. 1 Representative gust-velocity spectra.

Twelve hundred of the LO-LOCAT turbulence conditions were selected in a random fashion for this analysis. Spectra calculations were performed using the Fast Fourier Transform Algorithm computer programming technique. Longitudinal, lateral, and vertical gust-velocity spectra were computed in terms of spatial frequency. The spectra were normalized using the variance of the gust-velocity time function. The Nyquist frequency for all spectra was established at 50 cps so as to eliminate aliasing in the 0.04–10 cps frequency range of interest.

All spectra are being evaluated for noise content. Those spectra with questionable signal levels in relation to noise are eliminated from further analysis. In this analysis coherency functions between gust-velocity component pairs are used to determine if they are statistically independent and to provide a manifestation of data noise content. A typical spectra plot is shown in Fig. 1.

Representative Expressions

Experimentally determined spectra are being compared to turbulence spectra expressions suggested by Theodore von Kàrmàn,[2] J. L. Lumley and H. A. Panofsky,[3] H. L. Dryden,[4] and U. O. Lappe.[5] The expressions are as follows:

von Kàrmàn

$$\left[\frac{\Phi_v(k)}{\sigma_{tv}^2}\right]_K = \frac{L_{Kv}[2 + 377.5(L_{Kv}k)^2]}{[1 + 70.78(L_{Kv}k)^2]^{11/6}} \tag{1}$$

$$\left[\frac{\Phi_u(k)}{\sigma_{tu}^2}\right]_K = \frac{4L_{Ku}}{[1 + 70.78(L_{Ku}k)^2]^{5/6}} \tag{2}$$

Lumley-Panofsky

$$\left[\frac{\Phi_u(k)}{\sigma_{tu}^2}\right]_P = \frac{11{,}800}{C^2[1 + (2950k)^{5/3}]} \tag{3}$$

Dryden

$$\left[\frac{\Phi_v(k)}{\sigma_{tv}^2}\right]_D = \frac{L_{Dv}[2 + 6(2\pi L_{Dv}k)^2]}{[1 + (2\pi L_{Dv}k)^2]^2} \tag{4}$$

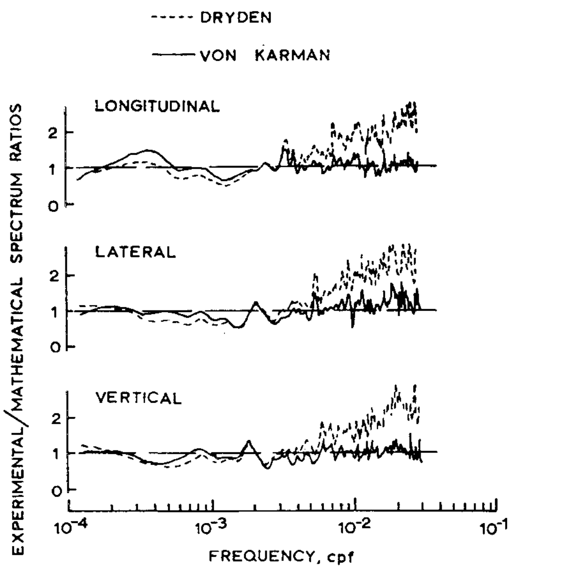

Fig. 2 Typical comparison of experimental spectra with von Kàrmàn and Dryden expressions.

$$\left[\frac{\Phi_u(k)}{\sigma_{t_u}{}^2}\right]_D = \frac{4L_{Du}}{1 + (2\pi L_{Du}k)^2} \tag{5}$$

Lappe

$$\left[\frac{\Phi(k)}{\sigma_t{}^2}\right]_L = \frac{2\pi L_L}{(1 + 2\pi L_L k)^2} \tag{6}$$

Equations (1) and (4) are intended to represent both the vertical and lateral components of turbulence and Eqs. (2, 3, and 5) are for the longitudinal component. The use of Eq. (6) has been suggested for all three components.

Good agreement has been found between the experimental spectra and the von Kàrmàn equations. Equation (3), provided by Lumley and Panofsky for the longitudinal gust-velocity component, is also giving a good representation of the experimental data. Equations (4–6) have not exhibited a good representation of the experimental data, especially at the higher frequencies. The basic disagreement has been in the turbulence inertial subrange where these expressions have a -2 logarithmic slope rather than the $-\frac{5}{3}$ slope of the data. This finding is pertinent, since Eqs. (4) and (5) have been used extensively in the past and are widely used today.

Typical comparisons are shown in Figs. 2 and 3. In each case, the experimentally determined spectrum values have been divided by values from the mathematical expression in question and these ratios plotted against frequency. Comparisons of the experimental spectra with the Lappe expression have given results essentially the same as those for the Dryden expressions.

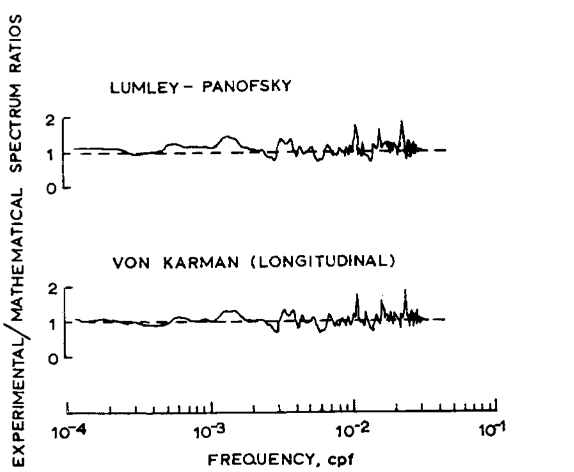

Fig. 3 Typical comparison of experimental spectra with the von Kàrmàn and Lumley-Panofsky expressions.

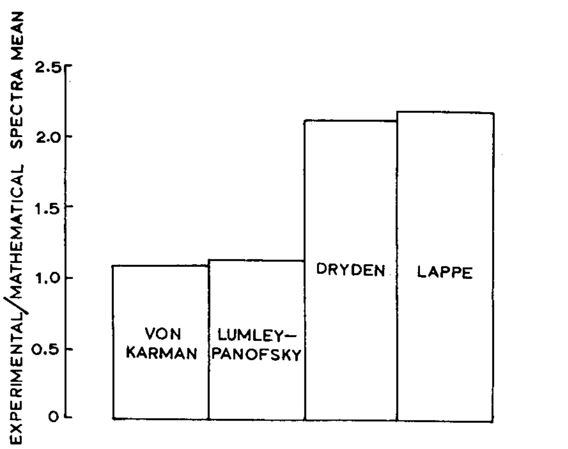

Fig. 4 Correlation of mathematical turbulence spectra expressions with experimental data.

Evidence of the consistent agreement between experimental data and the von Kàrmàn and Lumley-Panofsky equations is given in Fig. 4. Ordinate mean values for plots such as Figs. 2 and 3 were computed for a large number of conditions. These values were then averaged to give the results shown. The average mean values near 1.0 determined for the von Kàrmàn and Lumley-Panofsky expressions indicate good correlation with experimental spectra, whereas less agreement is shown for the Dryden and Lappe expressions.

The von Kàrmàn equations are based on isotropic conditions. LO-LOCAT experimental data have, in general, shown atmospheric turbulence at low altitude to be isotropic. Isotropy associated with each test condition is being investigated by dividing longitudinal and vertical gust-velocity spectra components by the corresponding lateral component. Theoretically, the vertical-to-lateral ratio should be equal to 1.0 under isotropic conditions. The longitudinal-to-lateral ratio should equal 0.75 in the high-frequency turbulence inertial subrange and increase with decreasing frequency approaching 2.0 in the low-frequency range. An example of the isotropic conditions being noted is given in Fig. 5.

Scale Length

The atmospheric turbulence scale length L is an indication of the average eddy size associated with the longitudinal turbulence component. Disagreements have occurred in the past concerning the scale lengths to be used in design applications. Estimations of this parameter have varied from 500 to

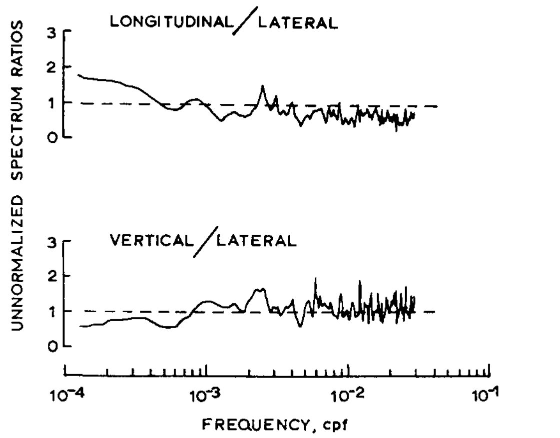

Fig. 5 Typical indication of atmospheric isotropy.

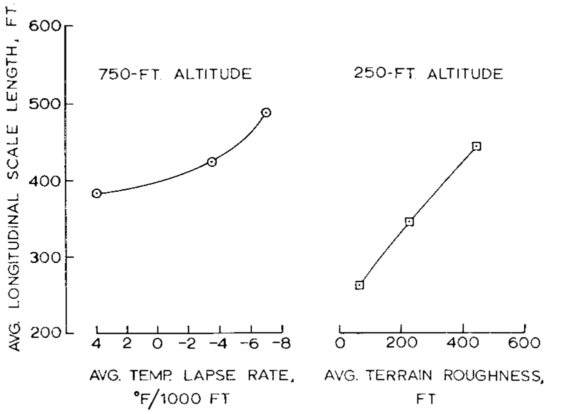

Fig. 6 Altitude, atmospheric stability, and terrain roughness effects on turbulence scale length.

5000 ft. In the LO-LOCAT Program, scale lengths corresponding to the von Kàrmàn expressions are therefore being analyzed statistically considering the geophysical conditions under which they are obtained. Scale length based on the von Kàrmàn expressions is represented by one of the following equations, depending on the gust-velocity component involved:

$$L_{Kv} = 0.110(\sigma_t/\sigma_T)_v{}^3(1/k_1{}^{2/3} - 1/k_2{}^{2/3})^{3/2} \tag{7}$$

$$L_{Ku} = 0.0717(\sigma_t/\sigma_T)_u{}^3(1/k_1{}^{2/3} - 1/k_2{}^{2/3})^{3/2} \tag{8}$$

Equations (7) and (8) were derived from Eqs. (1) and (2) considering a standard deviation of the truncated spectrum covering only the high-frequency range:

$$\sigma_T = \left[\int_{k_1}^{k_2} \Phi(k)\, dk\right]^{1/2} \tag{9}$$

The integration in Eq. (9) may be accomplished by noting that, in the truncated frequency range, $70.78(Lk)^2 \gg 1$, allowing simplification of Eqs. (1) and (2).

Analysis of the 1200 LO-LOCAT spectra indicated certain turbulence scale length trends. Longitudinal scale lengths were computed from vertical gust-velocity components using Eq. (7). At the 750-ft absolute altitude, scale lengths tended to increase with decreasing atmospheric stability, as shown in Fig. 6. At this altitude, the temperature lapse rate appeared to have a greater influence than terrain roughness. At the 250-ft altitude, terrain roughness appeared to be the major influencing factor. A trend of increasing scale length with increasing terrain roughness occurred as shown in Fig. 6. Also, scale lengths appeared to increase with altitude.

For all but the very long wavelengths, it can be shown that C in Eq. (3) is related to the scale length L in Eq. (2) as follows:

$$C = 0.415(L)^{1/3} \tag{10}$$

In Ref. 3, a value of 2.45 is suggested for C; LO-LOCAT data indicate an average value of 3.00.

The magnitude of atmospheric turbulence at low altitudes has been found to increase with decreasing atmospheric stability, increasing terrain roughness, and decreasing altitude. Cumulative probabilities of gust-velocity standard deviation values obtained thus far are shown in Fig. 7. The distribution is based on 5551 conditions involving all geophysical situations encountered with the C-131 aircraft. Additional testing is to be accomplished with a higher-speed aircraft in the near future. It is anticipated that somewhat higher values will be obtained over high mountains at the Colorado route during this phase of the program, since closer contour flight will be possible. Additional performance of the aircraft will also allow the analysis of turbulence wavelengths up to 14,000 ft in length.

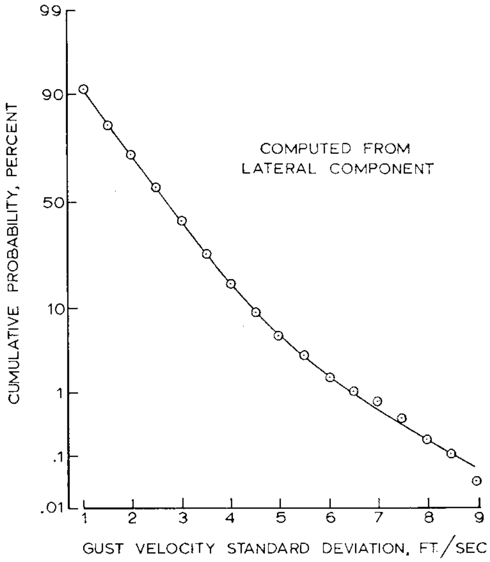

Fig. 7 Cumulative probability of gust-velocity standard deviation.

Conclusions

The LO-LOCAT research program has shown that, for wavelengths up to 7000 ft, the von Kàrmàn equations give a good representation of gust-velocity power spectra at low absolute altitudes. Atmospheric turbulence average longitudinal scale lengths at low altitude have varied from 259 to 500 ft depending on absolute altitude, atmospheric stability, and terrain roughness. Gust-velocity standard deviation values have ranged from 0 to 9 fps with a mean of 2.79.

References

[1] Gault, J. D., "Low Altitude Atmospheric Turbulence Analysis Methods," *Canadian Aeronautics and Space Journal*, Vol. 13, No. 7, Sept. 1967, pp. 307–314.

[2] von Kàrmàn, T., "Progress in the Statistical Theory of Turbulence," *Turbulence Classic Papers on Statistical Theory*, Interstate Publishers, New York, 1961, pp. 162–173.

[3] Lumley, J. L. and Panofsky, H. A., "The Structure of Atmospheric Turbulence," *Interscience Monographs and Texts in Physics and Astronomy*, Vol. III, Wiley, New York, 1964.

[4] Dryden, H. L., "A Review of the Statistical Theory of Turbulence," *Turbulence Classic Papers on Statistical Theory*, Interstate Publishers, New York, 1961, pp. 115–150.

[5] Lappe, U. O., "Design of a Low Altitude Turbulence Model for Estimating Gust Loads on Aircraft," AFFDL-TR-64-170, March 1965, Research and Technology Div., Wright-Patterson, Air Force Base, Ohio.